Last year project report

Risk Assessment Methods and Prioritization of Hydrographic Surveys

Student: Colonne Thomas

Project tutor: Seube Nicolas  School tutor: Moitié Rodéric
Acknowledgement

I would like to take this opportunity to thank all the people who have contributed in some way to this project. I would like to thank Mr Nicolas Seube who supervised and mentored my work, for his attention, and for all the time he awarded to me, his advices was really useful and appreciated. I also thank Mr Jean Laflamme, who gave me the opportunity to work at CIDCO for the second time. Thank to all the member of CIDCO who were always ready to help me, and thank Line bérubé and Coralie Monpert for their good humour.

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I will finally thank my parents. My father, for giving the will of work. My mother for believe in her children more than in herself.
Summary:

This report present the study made by CIDCO on risk assessment methods and prioritization of hydrographic surveys in the Canadian Arctic area. We detail the development of maritime corridors, routes and the associated requirements in terms of marine charts.

We first present three existing studies on this subject. The first two, proposed by the Arctic Region Hydrographic Commission (ARHC) and the Canadian Hydrographic Service (CHS), deals with the arctic case and studying bathymetric issues and marine corridors concept. The third one has been developped from the Land Information New-Zealand (LINZ). It define a risk assessment method using a risk matrix concept and compile the result with a economic cost/benefit study in order to prioritize surveys area in the south west Pacific.

According with the conclusions of our analysis of the previous studies, we explain a new methodology for hydrographic surveys prioritization. This methodology match up concepts from previous studies such as LINZ risk assessment method and CHS marine corridors concept.

Sommaire:


Nous présentons trois études qui ont été menées sur ce même sujet. Deux d’entre elles proviennent de l’ARHC (Arctic Region Hydrographic Commission) étudient la zone Arctique et les risques lié à la bathymétrie générale et au trafic afin de définir le concept de corridors maritimes. La troisième vient du LINZ (Land Information New Zealand) et définit une méthode d’évaluation du risque basé sur le concept de matrice de risque. Le résultat de cette matrice sera ensuite couplé à une étude économique de cout/bénéfice afin de prioriser les levés hydrographiques dans la zone sud ouest Pacifique (plus spécifiquement la zone des Tonga).

D’après les conclusions de notre analyse de ces études, nous expliquons notre nouvelle méthodologie pour la priorisation des levés hydrographiques. Notre étude recoupe les différents concepts étudiés comme l’évaluation du risque du LINZ et le concept des corridors maritime.
## Glossary

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1. Introduction

When navigating a ship, the sole indication on the sea-floor comes from the nautical chart, which is compiled from hydrographic data. Whenever hydrographic data are submitted to a certain level of uncertainty, the nautical chart may depict sea-floor information which is not in accordance with the reality. This type of situation may lead the mariner to take inappropriate decisions.

Representing uncertainty in Nautical Charts, in order to help the mariner in his decision making is an avenue on which number of Hydrographic Services are working around the world.

Nautical charts may become unreliable for the purpose of safety of navigation due to numbers of reasons, among them are:

- Evolution of traffic (ship traffic evolves in an area which was not originally surveyed and chartered for this purpose);
- Sea-floor dynamics (sand waves, effect of Tsunamis, etc...);
- Old-dated hydrographic data;
- Difficulty to assess the confidence in cartographic and hydrographic data

In the recent years, beyond the notion of data quality (related to IHO standards such like S-44 for hydrographic data and S-57 for cartographic information), the notion of risk assessment has gained in interest from the hydrographic community. Risk analysis admits the fact that nautical charts may not be conservative enough or may contain a high degree of uncertainty leading to possible groundings. The aim a risk assessment method is to relate the navigation risk to sources of errors that can be found in the charting compilation process and the associated hydrographic survey data quality factors.

Instead of defining a hydrographic and/or cartographic data quality criterion, the goal of risk assessment is to define, characterize and compute the navigational risk due to actual hydrographic data and charting information.

Generally speaking, the notion of hydrographic risk can be defined (following a frequency expectation loss approach) by the expectation of vessel grounding, weighted by a measure of its consequences.

Risk assessment can be viewed as the application of modeling techniques that can be used to capture all uncertainty factors due to hydrography, and relate them to a vessel grounding frequency analysis using existing or predicted traffic data.

Once risk assessment models are set, one can define the reward cost of vessel grounding consequences, and thus infer the risk factor as the product of the probability of grounding by the cost of related consequences. Hydrographic risk analysis should therefore focus on multiple uncertainty factors that may cause vessels grounding and also on the multiple consequences due to grounding.

One problem is to weight these multiple uncertainties and consequences costs, which is actually the main design problem of prioritization methods. Indeed, the weighting policies of both source factors uncertainty and consequences cost are left to the designer of the risk assessment system.
Therefore, a prioritization process highly depends on the data and expertise used to derive these weights (and in particular on econometric modeling of consequence costs). Therefore, designing an objective hydrographic survey prioritization tool requires the incorporation of well documented data, models and spatial analysis methods to produce results which can be well accepted by the stakeholders.

In this report we shall distinguish two types of risk assessment approaches:

Risk matrix development, following a methodology inspired by the IMO Formal Safety Assessment (FSA), mainly developed by the LINZ (Land Information of New Zealand). This approach is a global approach in the sense that to define the risk at a given location, a “risk matrix” weights some high level information, which can be assimilated to survey meta-data. Several criterion composing the risk matrix are ranked, all of them being defined spatially, and therefore managed by a Geographical Information System through overlay analysis. The core of this type of method is the determination of the relative weighting between each overlay defining a component of the navigational risk which has a significant impact on the overall risk assessment method.

Risk assessment via under-keel clearance models make use of a comprehensive information at a given location (hydrographic data, tidal information, navigational information) and aims at computing a probability of grounding. These models take into account bathymetric information (but not charting information) such like soundings, tidal models, navigational parameters (draft, heave) and can be used to assess the risk as a probability of grounding multiplied by the cost of the associated consequences, depending of the severity of the grounding, the grounding location, current information, distance to sensitive areas and the type of vessel.

Under-keel clearance model have been used in order to assess the navigational risk in local areas, mainly in harbors, channels and access channels while the purpose of risk assessment at a sub-regional scale. Therefore these two classes of methods seem not to follow the same objective and do to require the same type and amount of data in input.

A full description of available bathymetry and its associated level of uncertainty, tidal models is required for under-keel clearance models. Chart quality attributes (as defined in the IHO S-57 Standards), proximity to sensitive areas, type of navigation required are the type of data which should be populated in GIS overlays to apply a “risk matrix” approach.
2. Context

2.1. Shipping lanes in the Arctic

Receding sea ice extent in the Arctic, due to a changing climate, has recently opened up new shipping routes. These uncharted lanes see their traffic density increasing every year. Tourism, cargo shipping, oil terminal projects, fish ships Canadian Arctic is a high potential region in a lots of domains. The next figure shows the marine traffic in Arctic for the year 2011-2013:

![Figure 1: Shipping lanes in the Canadian Arctic region](image)

According to this figure, two major routes are emerging. The first one (in blue) connects the south of Baffin Island to the Hudson Bay, especially the port of Churchill. This port is the only one which has the infrastructure capacity for Panamax ship type. Currently, oil terminal projects are being in this port which confirms the need of high quality marine charts to avoid possible oil spill. The other important shipping lane is the North-West passage (in red) which has a huge economic potential: 4000km less than the Panama Canal route for the Europe-Asia transit, Mining projects and tourism.

This opening does not only concern oil and raw material transportation. Cruise industry and fisheries sectors also known a great development. So in addition to the environmental risk issue, the risk of loss of life is increasing to. The next figure shows us the number of ships travelling in the Canadian Arctic from the period 1974-2007.
According to this section, it is clear that the Canadian Arctic seas are getting open. Ships can travel further and sooner. Traffic density, loss of life and pollution potential are increasing. Canadian state, via the CHS, has to ensure the navigational safety by updating or create marine charts.

### 2.2. Overall state of cartography

Due to particularly hard surveying conditions, hydrographic and cartographic state does not ensure the quality level required. Most of data came from ante-1970 surveys. Soundings have been measured by lead lines, are geo referenced by legacy and inaccurate positioning system data.
This figure shows us the 2013 state of hydrographic data quality. Notice that the two major route highlight previously (Hudson Bay and North-west passage) only get CATZOC C attribute. In addition, most of ports approach areas does not reach the required CATZOC A.

On the cartographic state, according to the C-55 publication (September 2013), Canadian state gives the following information:

<table>
<thead>
<tr>
<th>Marine Cartographic State</th>
<th>Small Scale, Deep Sea Shipping Percent</th>
<th>Mesoscale, Coastal Navigation Percentage</th>
<th>Large Scale, Harbour Access Percentage</th>
<th>Marine Chart Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100</td>
<td>75</td>
<td>75</td>
<td>Paper</td>
</tr>
<tr>
<td>B</td>
<td>100</td>
<td>75</td>
<td>100</td>
<td>RNC</td>
</tr>
<tr>
<td>C</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>ENC</td>
</tr>
</tbody>
</table>

Table 1: Canadian Arctic cartographic state

According to this table, there is a huge proportion of available marine chart comparing to the low hydrographic data quality. We can conclude that a large data available on marine charts came from vintage surveys and with low quality and there are huge zones on these charts which do not contain any sounding.

To conclude this section we will come back on three important points:

- Marine traffic in Canadian Arctic Seas is increasing, for all navigation purposes (freight, cruises, fishing).
- The need for hydrographic and cartographic data is huge, and existing data do not match the quality required.
- These previous points raise navigation safety issues for human, economic and environmental reasons.

Considering the urgency and the importance of need for data, a hydrographic prioritization process is essential. We will present now different studies made on this subject.

3. Risk Assessment methods

Risk assessment methods has been develop in many areas in order do understand causes and consequences of risked events. These formal studies began in the nuclear industry, where risk control was essential. Since then, lots of risk assessment methods and tools have been developing for specific uses in risked industries and organisations.

In this section we will describe and give clear definitions of important elements, and an overview of existing risk assessments methods.
3.1. Definitions

The following definitions are not universal’s. They can change with the nature of the study.

- **Risk**: Product of the frequency and consequence of an event.
- **Frequency**: Measure of the occurrence (actual or probabilistic) of an event. Frequency can be derived from statistics study (absolute frequency) or from an estimation of the likelihood of an event occurring (subjective frequency).
- **Consequence**: Can be positive or negative (generally in a case of an accident). It can be express in terms of “most likely” and “worst credible”. It also can be measure with the financial cost of an event.
- **Events**: Describe an unplanned occurrence with consequential harm.
- **Risk analysis**: Systematic use of information and expert judgment to identify hazards and estimate their risks to people, property, environment and stakeholders.
- **Risk evaluation**: Establishing the tolerability level of a risk and an analysis of risk control option.
- **Risk assessment**: Risk analysis and evaluation.

3.2. Risk analysis methods

According to these definitions, lots of different methods have been develops for different problems. We will quickly describe here some of these methods. This list is not exhaustive, many more methodologies exists and are well document.

3.2.1. Coarse Risk Analysis

This method only presents a risk picture with relatively modest efforts. Risk, hazards, causes and consequence are quickly assessed by an expert judgment. Results are present on a graph format which present different argument of probability and consequence. Another graph will present different categories of consequence: risk to people, property, environment etc.

3.2.2. Hazard and Operability Studies

HAZOP is a qualitative risk analysis method which tried to identify facilities design weaknesses and hazards impacts if they are realised. The process studies causes and consequences in a case of a design intent failure. It is used in some sectors of the oil and gas industry to review and sequential operations to ensure appropriate safeguards systems.

3.2.3. Structured What-if Technique

SWIFT technique is lead by the question “what if?”. It is used to identify deviations from normal conditions. SWIFT is a brainstorming technique where personnel familiar with the system under examination identified possible problems, combination of condition which can create a risk. It is a preliminary technique and it is usually used before a most in deep HAZOP process.

3.2.4. Risk analysis methods in the maritime industry

International Maritime Organization, IMO, define a risk assessment analysis for shipping safety in 1997. Formal Safety Assessments (FSA) process is a “rational and systematic process for assessing the risks relating to maritime safety and the protection of the marine environment and
for evaluate the costs and benefits of IMO’s option for reducing these risks”. This process follows five steps:

- **Hazard identification**: definition of the present possible hazards according to the studying case. It usually involves collisions, fire, and grounding.
- **Risk analysis**: cause, frequency and consequence analysis.
- **Risk control options**: definition of possible countermeasures and their effects on risk causes, frequency and consequences.
- **Cost/Benefit assessment** of these risk control options.
- **Recommendations** for decision making.

This method is comprehensive and can be adapt in a lot of case of study. For hydrographic surveys prioritization and navigational risk assessment issues, the FSA process can meet our demand. By seeing hazards as possible causes of grounding, risk analysis as defining risk level of areas, and risk control option as marine charts production.

Different studies tried to answer these issues; we will present them in the next section.

### 3.3. Risk assessment for hydrographic survey prioritization

#### 3.3.1. ARHC:

The ARHC (Arctic Regional Hydrographic Commission, OHI), which regroup the United States, Norway, Denmark, Canada and Russia, presents its work on risk assessment at the 7th meeting of the IRCC at Mexico, in June 2015.

Their identification of survey priorities is based on three fundamental data sources: confidence of existing hydrographic data, water depth, and density of marine traffic. These three data can be considered independently on a low-to-high risk table:

<table>
<thead>
<tr>
<th>Data type</th>
<th>Relative Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence of Hydrographic Data</td>
<td>Low</td>
</tr>
<tr>
<td>Water Depth</td>
<td>High</td>
</tr>
<tr>
<td>Density of Traffic</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>High</td>
</tr>
</tbody>
</table>

Table 2: ARHC risk table

#### 3.3.1.1. Hydrographic data state

The level of confidence of hydrographic data is derived from different factors: acquisition equipment used, vintage of the survey data and surveying technique employed. CATZOC attribute categorize the accuracy of American and Canadian surveyed region due to a difference in data sources. Figure 4 is a sample visualization of the compilation of these different information on the eastern side of Bering Strait:
3.3.1.2. Bathymetric component

The next layer in their analysis is the water depth. From the International Bathymetric Chart of the Arctic Ocean (IBACO), they separate them in three classes: Shallow, Mid-depth and Deep. But they also take care of the seabed complexity. The navigational risk seems higher in a complex seabed where the depth can jump from 100 meter to 10 meter in a short time than in a simple area where the seabed depth stays at 25m. The idea is to separate the seabed complexity in two groups, simple and complex, using the Southern Alaska Coastal Relief Model. Then depths limits for Shallow, Mid-depth and Deep classes will change if the seabed is complex or simple. The next figure gives a visualization of results in the previous area.

Figure 4: Hydrographic data state computation over a studying area

Figure 5: Bathymetric depth classification
These two map layers are intersecting to delineate the areas of potential concern. Areas of potential concern are ranked from low to high based on their potential for navigational risk.

![Image of map layers intersecting to delineate areas of potential concern]

**Figure 6: Combination of hydrographic data state and bathymetric classification score**

### 3.3.1.3. Marine traffic Analysis

The third data source came from the AIS data base and represents the marine traffic. Indeed a high concern area with no vessel traffic will not be a huge priority. AIS data used spanned a time frame of one year between June 2012 and July 2013. It also has ship type information and permit to denote a “higher consequence” tracks lines. This vessel subset is based on the potential for loss of life, property and environmental integrity.

Then, traffic is intersected with the area potential concern map layer and gives “risked areas” location. This prioritization approach is explained by the following flow diagram:

![Flow diagram of ARHC complete algorithm]

**Figure 7: ARHC complete algorithm**
The ARHC method gives high priority to relatively shallow areas, with poor confidence hydrographic data, which are heavily transited.

To conclude, the ARHC developed a clear procedure to highlight risked areas. The risk is based on three components and is assess by a GIS tool. Their method is simple and gives good results. But their definition of risk is not comprehensive enough. They do not define any navigational risk. In addition, a cost benefit analysis would be important in the prioritization process, but they do not mention any.

3.3.2. CHS

The Canadian Hydrographic Service (CHS) also worked on the prioritization of hydrographic survey areas. Their method is based on the definition of marine corridors. Complete area coverage is impossible, so they create corridors where vessels would be able to navigate with a complete knowledge of the sea bed.

The corridors have been defined as follow:

- **Primary**: Major well known routes to access other corridors. High traffic density
- **Secondary**: Routes to communities. Medium or low traffic density.
- **Tertiary**: Routes to North Warning System sites and places of refuge. Medium and low traffic density.
- **Fourth**: Routes to existing resources development sites. Low traffic density.
- **Fifth**: Routes to proposed resource development sites. Very low traffic density.

Then, by using a GIS tool, CHS compute information such as:

- Tide and current.
- Environment data.
- Populated place.
- Port tonnage.
- Safety zones.
- Resources development and projections.
- CHS charts.
- Hydrographic data.

By combining these information, they can geo-positioned the different marine corridors. The next figure shows us the location of these corridors:
Figure 8: Marine corridors geo-localisation

Notice that primaries corridors are located on the two major routes presented previously: access to Churchill Port and the North West passage.

With corridors placed, the next step is to compute the "Level of Effort" for risk managing costs based on various hydrographic options. This step evaluates the cost of work to be undertaken for ensuring a level of quality.

Then a priority is given to corridors areas where water depth is less than 50m, with a complex seabed and where under keel clearance concerns would naturally drive the need for products supported by high resolution bathymetric data source. So, primary corridors, within deep water and non complex seabed would not necessary have a high priority. A template for modern bathymetric source data coverage requirements was proposed as follows:

- 0-50m water depth to be covered by CATZOC-A surveys.
- 50-100m water depth to be covered by CATZOC-A surveys where seabed is complex or CATZOC-B surveys where seabed is non-complex.
- 100-200m water depth to be covered by CATZOC-B where seabed is complex or CATZOC-C where seabed is non-complex.

The complex/non-complex classification of the seabed is derived from the General Bathymetric Chart of the Oceans (GEBCO).

The prioritization process is based on a difference between the available and the required hydrographic data. It permits to highlight areas which really need a hydrographic survey. Then, considering that ports and port approaches are high priority areas, a second-phase prioritization is performed on a port by port basis. This prioritization is based on risk matrix whit the following factors: Population traffic frequency, tonnage, the extent of approach area where water depths are less than 50m and seabed complexity.
To conclude on this study, the CHS regroup a lot of information in their GIS tool. It is able to defines and to geo-localised marine corridors. This solution permits to:

- Reduce the surveys areas.
- Regroup traffic on well surveyed area.
- Concentrate their level of effort on small area. For hydrographic surveys but also for coast guards, rescue and icebreaking services.

Their prioritization process is however based on few criterion: bathymetric consideration and quality level required. The combination of all these components does not clearly define a risk notion and are not enough comprehensives.

3.3.3. LINZ:

Land Information of New-Zealand also has prioritization issues. They had driven a study base on FSA process and risk matrix concept for risk analysis.

Risk has a lot of different definitions, and it can be assess by many different ways. Here they define it as the combination of a frequency and a consequence of an event. A risk matrix tends to express the presented definition of risk. The next figure shows a risk matrix. For a given event, the risk score is defining by its probability (from Very Low to Very High) and its impact (from Very Low to Very High).

![Risk Matrix](image)

**Figure 9: Example of risk matrix. Risk increase with the frequency and consequence.**

Here, the event is grounding. Its frequency cannot be only issued from historical data, and grounding probability depends on multiple inputs (meteorological, charts quality, human). Consequences (or impacts) of groundings also have several aspects: economical, human, and environmental. They need to define probability and consequences of a grounding event. But the final result will be the same: risk score is a combination of frequency and consequence mark.

LINZ sees grounding frequency as the combination of likelihood risk criteria (type of navigation, accuracy of nautical charts…) and an expression of the traffic. By combining it, a frequency can be given. Grounding event impact will be seen as a combination of different possible consequences (pollution of an important environmental area for example). After these definitions, a mark (from 0 to 5) is given to each criterion. Mark system can change from one criterion to another. It can be for example a level of proximity of an important economic site, or CATZOC attribute for the charts quality criterion. Then a weight will be applied to each criterion in order to express the importance of it in the entire model. The last step is to compute all criteria marks and weighs to express first the frequency and consequence marks, and then a risk level. The next table explains this process:
### 3.3.3.1. Risk Matrix criteria

Each criterion is defined by a risk assessment expert. It tends to express specific navigational hazards and grounding consequences in the studying area.

Weights system is arbitrarily define. It is supposed to express the importance of a criterion in the system. By evaluate them by a expert, they involves subjectivity and so errors in the risk result.

**Marine traffic**

Traffic matrix risk section is derived from a marine traffic analysis. Lots of studies go on this subject with several results. Some of them highlights shipping lanes and define marine corridors (categorize by their purpose and traffic density) [Canadian Coast Guard, 2014]. Here, the LINZ traffic section needs to be clear and summary. The two important component and possible consequence of grounding is the loss of life and pollution. LINZ chose to divide its traffic analysis in these two criteria:

- Potential loss of life.
- Pollution potential.

**Likelihood risk criteria**

This section regroups navigation hazards and elements which can involve grounding. Criteria will change from an area to another. Indeed, navigational hazards in the Pacific Ocean are not the same as in the Canadian Arctic Sea. But some of them can be used in any zones.

- Sea floor morphology.
- Proximity to the 15m isobaths.
- Current and meteorological conditions.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Marks</th>
<th>Weights</th>
<th>Traffic: $\sum_{i=0}^{3} M_i \times W_i$</th>
<th>Frequency: Traffic * Likelihood</th>
<th>Risk Mark: Frequency * Consequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traffic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>$M_1$</td>
<td>$W_1$</td>
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<td>Criteria 2</td>
<td>$M_2$</td>
<td>$W_2$</td>
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<td>Criteria 3</td>
<td>$M_3$</td>
<td>$W_3$</td>
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<tr>
<td>Likelihood Risk Criteria</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Criteria 8</td>
<td>$M_8$</td>
<td>$W_8$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consequence</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Criteria 9</td>
<td>$M_9$</td>
<td>$W_9$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Criteria 10</td>
<td>$M_{10}$</td>
<td>$W_{10}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Criteria 11</td>
<td>$M_{11}$</td>
<td>$W_{11}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Criteria 12</td>
<td>$M_{12}$</td>
<td>$W_{12}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Criteria 13</td>
<td>$M_{13}$</td>
<td>$W_{13}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Risk matrix concept
- Hydrographic and cartographic data state.
- Proximity to known shoals

In the LINZ study they add proximity to known WW2 military sites, and to volcano for example. These criteria are specific to the South Pacific and may not be relevant in Canadian Arctic seas.

**Consequences**

Consequences also depend on the studying area, but it can be categorized in three sections: economic, environmental and cultural. Consequences criteria are usually proximity of an important site and define by an expert judgment. LINZ defines for example:

- Proximity to Large Reef
- Proximity to world Biological Protected site
- Proximity to Local cultural protected or important sites

As the likelihood criteria, consequences need to be defining locally. A group of expert will choose which type of area seems important or sensitive to pollution.

**3.3.3.2. Cost benefit assessment**

After assessing navigational risk level of the different studying areas, LINZ applies a cost benefit method in order to monetarize hydrographic surveys. They are studying their cost and their benefit for economical area. They are also studying the cost of a non survey. If an area stays unsurveyed for too long, the grounding probability will increase, as well as the potential cost of grounding.

This study takes part in the decision making process. It helps to organize the future hydrographic surveys. Hydrographic services had to ensure the marine charts quality, but they also need an economic study.

**3.3.4. Discussion on these studies:**

**3.3.4.1. Weighted overlay analysis**

All of these studies have the same approach. They use GIS system in order to combine different information on the same zone. Each criterion will be assigned a mark from “lower risk” to “Higher risk”. Then the combination of these criteria will give a risk map for the entire zone.

![Weighted Overlay Analysis Diagram](image)

**Figure 10: Weighted overlay analysis principle**
It permit to combine quickly an easily data which came from different source and which have different nature. With such wide studying areas, weighted overlay analysis is the most efficient way to get a risk terrain model.

### 3.3.4.2. Risk definition

The notion used for risk is important for prioritization studies. Indeed a risked area has priority over a non-risked one. In the LINZ study, the risk is clearly defined as a combination of the frequency and consequence of an event. The ARHC and CHS express it only with bathymetric and traffic consideration.

A shallow and complex seabed will had a higher risk over a deep and non complex one. This is not a clear definition of likelihood of grounding. In addition, they do not consider others criteria, such as meteorological or currents conditions, as likelihood risk criteria. However, charting risked areas (high currents etc) will permit mariners to better plan their routes, and avoid possible groundings.

Finally, with the combination of an AIS data processing; ARHC and CHS can express a frequency of grounding. But they do not clearly examine the consequences. CHS will give the priority to port approaches with a port by port analysis, which can be seen as a consequences examination. But, as the frequency, it is not clearly defined as possible grounding consequences.

### 3.3.4.3. Marine corridors concept

The creation of well surveyed marine corridors is firstly defined by the CHS and taken up by the ARHC. It permits to reduce the hydrographic survey efforts on smaller zones where ships are actually navigate. The LINZ risk matrix also refers to these corridors indirectly. Indeed the risk score will be null if there is no navigation in an area, and so they will be seen as lower priority. But it did not clearly define these areas as marine corridors.

The CHS marine corridors definition gives us, in addition to the traffic density information, a global purpose of the marine traffic. In the Arctic case, where some communities depends on supplies ships, purpose of navigation is important and should be taken in the risk assessment process.

### 3.3.4.4. Summary

ARHC and CHS methodologies are objectives approaches. Areas of concern are areas with poor hydrographic dataset, shallow water and high traffic density. The proposed prioritization process is drive by few and clear criteria. Hence their results are clearly objective. But their risk definition forgets lots of criteria, as current condition, and the most important: consequences of grounding.

The LINZ methodology is complete in the risk definition and computation. Traffic analysis, likelihood criteria, consequences are clearly defined. Then a cost benefit analysis is done in order to help expert in their decision making process. But, due to a large number of criteria, they need to quantify their importance in the model by a weights system. By doing it they involve subjectivity in risk matrix results.
4. Development of risk assessment methods

The previous section has shown us that a risk matrix approach seems the most consistent method in order to assess the navigational risk of an area. In this section we will present a possible risk matrix, adapted to the Canadian Arctic region. This matrix regroups the three sections: traffic, likelihood criteria and consequences.

The presented list is here not exhaustive and should be validated by a working group, gathering the different stakeholders (Hydrographers, Captains, economists, community’s representatives for example).

4.1. Risk matrix definition

A risk matrix design can be divided in three steps:

- Traffic analysis
- Navigational hazards identification
- Consequences identification

Each of these steps needs to be studied by a group of expert which will express all possible criteria and their weights.

We will here present a possible risk matrix for the Arctic region.

4.1.1. Marine traffic analysis

According to the previous studies, two different traffic analyses can be used:

- Marine corridors concept
- Potential of loss of life and pollution.

Marine corridors concept is defined in order to reduce surveys areas and to provide well charted shipping lanes. But, according to the Desgagné company, which actually navigate in the Arctic region: “Marine corridors concept is dangerous: it would limit captain’s navigational initiatives when they need to change their routes due to an ice blockage of the corridor”. Marine corridors will force ships to navigate in restricted lanes whereas navigation in the arctic is mostly based on captain estimations and initiatives.

In addition to this, to respect the risk definition, the traffic section needs to express a frequency. Therefore, criteria for traffic analysis will be restricted to traffic density. Notion of marine corridors will not be used here.

However, density can be expressed in many ways: number of vessel per years, subdivision per ship types and size. Here, regarding grounding consequences, two important topics will drive our analysis:

- Potential of loss of human lives.
- Pollution Potential.

The Traffic section will be hence divides in two sections: potential of loss of life and pollution potential.
4.1.2. Likelihood criteria

Likelihood criteria define possible causes of grounding. It can be divided in two sections: navigational hazards and aids to navigation state.

Navigational hazards

In this section, we list all possible causes of grounding. The following navigational hazards can be introduced:

- Sea floor morphology
  - Complexity
  - Bottom type
  - Proximity of known shoals
- Current and meteorological conditions
  - Current speed
  - Wind conditions
  - Visibility
- Ice coverage
  - Ice coverage period
  - Type of ice (soft, hard)

Navigation complexity and aids to navigation

Marine charts and buoys permit ships to avoid possible hazards. But, a poor condition or drift due to receding ice coverage of these aids will gives to the ship false information and can involve grounding. The following criteria represent this risk:

- Marine charts quality and age
  - CATZOC attribute
- Aids to Navigation (port approach buoys…)
  - Accuracy and conditions

Buoys can be, at the first place, not properly installed. It may drift by bad weather conditions or by the ice effect. A bad position of these aids will create a difference between the marine chart and the navigator visualization, which will introduce a risk.

According to S-57 specification, CATZOC attribute represent cartographic data state. But in the Canada area, some typical issues exist:

- Verticals references.
- Horizontals shifts.
- Poor tidal models.

These issues can be explained by the difficulty to get data in the North, but they are not always clearly define and may be concealed in the CATZOC attribution process. So, on areas where there is a doubt on hydrographic and cartographic data quality assessment, a further study needs to be done on:

- Age of surveys
- Type of instruments used
- Marine positioning system used
Type of navigation also is a source of risk. A port approach area is more risked than an off shore navigation.

- Type of navigation
- Distance to 15m contour

### 4.1.3. Consequences

Consequence criteria are essential in a risk assessment process. They represent the cost of grounding, pollution on a protected environmental area for example. We need here to list all possible consequences of a grounding. They can be dividing in three categories:

- **Economic**
  - Proximity to sites of high economic contribution
  - Proximity to sites of moderate economic contribution
  - Proximity to ports
  - Proximity to Tourism sites

- **Environment**
  - Proximity to protected area of high importance
  - Proximity to protected area of moderate importance

- **Cultural**
  - Proximity to communities

Proximity to communities also raises another problem. Some communities depends on supplies ships, which in some cases, only pass once a year. If one of these ships cannot supply the community, it will put it entire community in a dramatic situation. We need to express a criterion which represents the purpose of navigation. It will give a higher importance to type of traffic.

Marine corridors are defined by the traffic density and the purposes of navigation. We can use this information in order to create a new consequence criteria based on the type of marine corridor.

- **Marine traffic purpose**

The use of marine corridors concept is not here a restriction of hydrographic surveys on smaller areas but an expression of the traffic type and importance.

The next table present a proposed risk matrix for the Arctic region. It could be improved through a design process involving stakeholders (hydrographic services, shipping companies, communities, local government representatives,..).
<table>
<thead>
<tr>
<th>Traffic Risk Criteria</th>
<th>Traffic Impact</th>
<th>Risk Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traffic type</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Sea Floor Morphology</td>
<td></td>
<td>Insignificant</td>
</tr>
<tr>
<td>Seabed Complexity</td>
<td>Very Low Complexity</td>
<td>Low Complexity</td>
</tr>
<tr>
<td>Bottom Type</td>
<td>Soft</td>
<td>5-10nm</td>
</tr>
<tr>
<td>Proximity to Known Shoal</td>
<td>&gt;10nm</td>
<td>5-10nm</td>
</tr>
<tr>
<td>Meteorological Conditions</td>
<td>Prevailing Condition Exposure</td>
<td>Sheltered at most time</td>
</tr>
<tr>
<td>Spring Mean Current Speed</td>
<td>Open Sea</td>
<td>1-2 knots</td>
</tr>
<tr>
<td>Visibility</td>
<td>Unknown</td>
<td>Poor Visibility Very Unlikely</td>
</tr>
<tr>
<td>Ice Coverage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Period of coverage</td>
<td>Free at most time</td>
<td>Mainly Free</td>
</tr>
<tr>
<td>Type of Ice</td>
<td>Soft</td>
<td>5-10nm</td>
</tr>
<tr>
<td>Type Of Navigation</td>
<td>Open Sea&gt;10nm</td>
<td>Offshore Navigation (5-10nm)</td>
</tr>
<tr>
<td>Distance to 15m Contour</td>
<td>&gt;10nm</td>
<td>5-10nm</td>
</tr>
<tr>
<td>Aid to Navigation</td>
<td>CATZOC Attribute</td>
<td>A</td>
</tr>
<tr>
<td>Bu oys Conditions</td>
<td>Very Good</td>
<td>Good</td>
</tr>
<tr>
<td>Environmental Impact of Grounding</td>
<td>Proximity to Protected Area of High Importance</td>
<td>&gt;10nm</td>
</tr>
<tr>
<td>Proximity to Protected Area of Moderate Importance</td>
<td>&gt;10nm</td>
<td>5-10nm</td>
</tr>
<tr>
<td>Economical Impact of Grounding</td>
<td>Proximity to Economical Area of major importance</td>
<td>&gt;10nm</td>
</tr>
<tr>
<td>Proximity to Economical Area of moderate importance</td>
<td>&gt;10nm</td>
<td>5-10nm</td>
</tr>
<tr>
<td>Proximity to important infrastructure (Ports)</td>
<td>&gt;10nm</td>
<td>5-10nm</td>
</tr>
<tr>
<td>Proximity to Tourism Site</td>
<td>&gt;10nm</td>
<td>5-10nm</td>
</tr>
<tr>
<td>Cultural Impact of Grounding</td>
<td>Proximity to Community</td>
<td>&gt;10nm</td>
</tr>
<tr>
<td>Traffic impact</td>
<td>Marine corridor type</td>
<td>Fifth</td>
</tr>
</tbody>
</table>

Table 4: Proposed risk matrix for Canadian Arctic seas
4.2. Risk matrix calibration.

Risk matrix methods can integrate a large number of criterions and can express the navigational risk on large zones. But weights are often subjectively defined by an expert judgment. By doing so, frequency and consequence are subjectively scored, and as consequence, the risk result may be biased. In order to overcome the subjectivity of the weight determination, we propose to estimate the weights by comparing risk matrix results with an objective risk computation model, which will compute actual frequency and consequences of grounding.

Let us express the result from the risk matrix as computation of risk marks and weights:

$$ R = F \cdot C $$  \hspace{1cm} (2)

<table>
<thead>
<tr>
<th>Traffic</th>
<th>Marks $X_T$</th>
<th>Weights $W_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Likelihood</td>
<td>Marks $X_L$</td>
<td>Weights $W_L$</td>
</tr>
<tr>
<td>Consequence</td>
<td>Marks $X_C$</td>
<td>Weights $W_C$</td>
</tr>
</tbody>
</table>

Table 5: Weights independency

$$ R = (X_T^T \cdot W_T) \cdot (X_L^T \cdot W_L) \cdot (X_C^T \cdot W_C) $$  \hspace{1cm} (3)

The equation (3) express the risk result for an area, considering marks and weights as vectors and risk result as combination of these vectors. Weights are here considered as unknown and marks as known variables. Complete weights estimation will need to express the influence of all risk matrix criteria on the final result. But traffic section only has two criteria, weighting them do not required a mathematical estimation. In addition, consequence section would require expert judgment and estimation. As hydrograph experts we will only estimate the likelihood weights, in other words: the influence of sea bed, cartographic state and weather criteria on the risk result.

We consider now $X_T, W_T, X_C, W_C$ as constant. Equation (3) is now a linear one:

$$ R = \alpha \cdot (X_L^T \cdot W_L) $$  \hspace{1cm} (4)

The estimation will be easier. Appendix B details the development of this estimation; we will only present here the result:

The optimal weights $W_L$ are solution to:

$$ A \cdot W_L = B $$

Or, on its complete form:

$$ \left( \begin{array}{ccc}
\sum_j X_{1j}^2 & \ldots & \sum_j X_{1j} \cdot X_{nj} \cdot \frac{1}{2} \\
\vdots & & \vdots \\
\sum_j X_{nj} \cdot X_{1j} & \ldots & \sum_j X_{nj}^2 \cdot \frac{1}{2}
\end{array} \right) \cdot \left( \begin{array}{c}
W_1 \\
\vdots \\
\frac{W_n}{\lambda}
\end{array} \right) = \left( \begin{array}{c}
\sum_j X_{1j} \cdot \eta_j \\
\vdots \\
\sum_j X_{nj} \cdot \eta_j
\end{array} \right) $$  \hspace{1cm} (5)
Where:

- $X_{ij}$ is the mark on criteria $i$ and risk matrix scenario $j$.
- $r_j$: Risk matrix result combined with traffic and consequence marks and weights on scenario $j$ (see appendix B).
- $\bar{W}_j$: Weights to be estimate.

We can notice two important points:

- First that the matrix $A$ looks like a Variance/Covariance matrix. We actually express the influence of criteria on risk result.
- Secondly, the number of scenario $j$ is important. The more scenarios different we have, the better we will see the influence of the different criteria and so the more general estimated weights will be. Let us take an example: if we only estimate weights on a soft sea floor, these weights will not be accurate on a hard sea floor case. Therefore, if we want general weights, we will need to simulate a wide range of all possible scenarios.

According to these be able to estimate, in order to estimate likelihood weights, we need to express an objective model which will compute a risk result for all possible scenarios. We will present in the next section one of these models and its application.

### 4.2.1. Under-keel clearance models and risk assessment

Weights estimation will be done in order to reduce the subjectivity part of the risk matrix weight determination. We therefore need to find an objective risk model which follows the two following characteristics:

- It gives values which have the same nature than the risk matrix outputs.
- It should be as objective as possible.

As we already mentioned in the previous section, we will only estimate frequency weights. So our grounding model will need to compute the actual grounding frequency following different scenarios.

#### 4.2.1.1. Preliminary definitions

Let us recall some basic definitions.

Let’s start by hydrographic data, which comprises soundings, tidal information, current information, and type of sea-floor.

A *sounding* represents the available navigable depth on a given location, with respect to a vertical datum. The location of a sounding is coordinated in a map projection frame with respect to a given Horizontal Datum, relative to a chosen geodetic system. The vertical datum is generally the Chart Datum (CD), which is defined in relation the lowest astronomical tide level. Within a relatively small area the CD can be considered as a constant, but its variation with location may be significant in coastal areas.

Depth measurements can be given from a wide variety of instruments, including legacy instruments: lead lines, Single-Beam Echo-Sounders, Multi-Beam Echo-Sounders and Bathymetric...
LIDAR. At a given location depth measurements are reduced from the tidal water level to provide sounding, this is to say the distance from the sea-floor to the Chart Datum.

The sounding uncertainty is thus depending on depth measurement, position, tidal information, and vertical reference uncertainties.

**4.2.1.2. Under-keel clearance**

The Under-Keel Clearance (UKC) of a vessel is the available depth between the lowest point of the keel and the sea-floor at a given location \( P \) and a given time \( t \). It will be denoted hereafter by \( u(P, t) \).

\[
u(P, t) = Z(P) + T(P, t) - (d + h(t))
\]

- \( u(P, t) \) is the under-keel clearance. It depends on the vessel position \( P \) and on the time \( t \).
- \( Z(P) \) is the sounding at the location \( P \). The sounding given by the sea chart is defined by the distance between the vertical Chart Datum (CD) and the sea floor.
- \( T(P, t) \) is the water level at position \( P \) and time \( t \).
- \( d \) is the vessel’s draft including navigational clearance. It does not depend on the time \( t \) neither on the location \( P \).
- \( h(t) \) is the vessel’s heave at time \( t \).

Let us note that this definition depends on two types of parameters:

- Vessel’s navigation parameters: heave \( h(t) \) and vessel’s draft \( d \). Heave depends on the sea state and the type of vessel. The draft is known from vessel’s loading information and is supposed here to include navigation clearance.
- Hydrographic information: Water level, soundings and Chart.

**4.2.1.3. UKC error analysis.**

The definition of under-keel clearance underlies different sources of error, as it depends on sounding, tidal information, and navigational information. In this section we detail the sources of error for all the model components. A synthetic UKC uncertainty model will then be derived.
Sources of error

- Draught uncertainty: It static component may be due to measurement errors after vessel’s loading. A dynamic component may be due to the squat effect (static negative heave motion in case of low UKC). This effect is particularly sensitive in ports and shallow access channels.
- Heave uncertainty: This component is directly linked to the sea state and will differ for each class of vessel.
- Position P: May be corrupted by positioning errors, and also by a static component which depends on the horizontal chart datum shift.
- \( Z(P) \): The sounding locate at the position \( P \) admit an error due to the position uncertainty and a depth error.

4.2.1.4 The UKC uncertainty model

Probabilistic study

As detailed in Appendix A, an under-keel clearance uncertainty model can be derive from an error analysis and the propagation of errors from the various components of the UKC model. It can be shown that if sounding, tide models, position, draft and heave follows normal laws, then the under-keel clearance, as a linear combination of those components also follows a normal law, which most probable estimate and variance are as follows:

\[
u(P, t) = Z_{obs}(P) + T_{obs}(P, t) - (d_{obs} + h_{obs}(t))\]  \(\text{(7)}\)

\[
\sigma_{\text{ukc}}^2 = \frac{\partial Z_v}{\partial E}(E_0)^2 \sigma_{\text{E}}^2 + \frac{\partial Z_v}{\partial N}(N_0)^2 \sigma_{\text{N}}^2 + \sigma_{Z(P)}^2 + \sigma_{ST(P)}^2 + \sigma_d^2 + \sigma_h^2 \]  \(\text{(8)}\)

Where:

- \((\frac{\partial Z}{\partial E})^2 \sigma_{\text{E}}^2 + (\frac{\partial Z}{\partial N})^2 \sigma_{\text{N}}^2\): is a term which depends on the expected or estimated terrain morphology. It should be noticed that this term can only be defined locally, in other words, terrain morphology can’t be extrapolated for significant positional errors.
- \(\sigma_{Z(P)}^2\): sounding uncertainty;
- \(\sigma_{ST(P)}^2\): tide uncertainty;
- \(\sigma_d^2\): draft uncertainty;
- \(\sigma_h^2\): heave uncertainty.

UKC model will follows a normal law centred on \(u(P, t)\) with an uncertainty of \(\sigma_{\text{ukc}}^2\). Hence, probability of the event “grounding” (denoted by \(G = \text{true}\)) is linked to the under-keel clearance by

\[
Prob(G = \text{true}) = Prob(u(P, t) < 0) \]  \(\text{(9)}\)

\[
Prob(G = \text{true}) = Prob(u(P, t) < 0) \]  \(\text{(11)}\)

- 28 -
In a similar way, one can define the probability of grounding at $k$ meters height by

$$\text{Prob}("G = k") = \text{Prob}(u(P, t) = -k)$$  \hspace{0.5cm} (13)$$

We notice that the probability of grounding, as the under-keel clearance is defined locally in space and time.

With a position $P$, one can define the risk of grounding as the product of the probability of grounding and the cost of the consequences linked to this event. One can therefore define the risk of grounding at position $P$ by

$$r(P) = \int_{k=-\infty}^{0} \text{Prob}("G = k") \cdot C(P, k) \, dk$$  \hspace{0.5cm} (15)$$

Where $C(P, k)$ denotes the cost function associated to the event “grounding at position $P$ with $k$ meters”. Notice that this cost function may be computed from grounding consequence models for different class of vessels (see for instance [Sven 2002]).

**MinMax analysis**

Another way to compute UKC model without any probabilistic model is to study a “worst case”. Instead of seeing each UKC model component as following a normal law, we will consider them as constant, but still submitted to uncertainty.

- **Sounding $Z(P)$**: let us consider $\Omega_z$ the set of sounding included in the position uncertainty ellipse of the vessel.

  $$Z_{\min} = \text{Min}(\Omega_z) + \delta z_{\max}$$  \hspace{0.5cm} (17)$$

- **Tide $T(P, t)$**: 

  $$T_{\min}(P, t) = T + \overline{\delta T_{\max}}$$  \hspace{0.5cm} (19)$$

- **Draft $d$**: 

  $$d_{\max} = d_{\text{true}} + \delta d_{\max}$$  \hspace{0.5cm} (21)$$

- **Heave $h(t)$**: heave can be seen as a sinusoidal function. Here we will see it as a constant (sinusoidal function amplitude).

  $$h_{\max}(t) = h_{\text{true}}(t) + \delta h_{\max}$$  \hspace{0.5cm} (23)$$

Each UKC component represents the worst possible value for navigation safety considering all uncertainties and errors. The UKC result, for a vessel, at position $P$ and time $t$, will express the worst possible scenario.

$$u(P, t) = Z_{\min} + T_{\min}(P, t) - (d_{\max} + h_{\max}(t))$$  \hspace{0.5cm} (25)$$

Notice that this under keel clearance definition gives us a constant, not a normal distribution. This facilitates the model computation over a ship trajectory.
The sector around each vessel location is the set of possible vessel positions considering possible marine chart horizontal shift, meteo-oceanographic conditions, navigator decisions, and manoeuvrability constraints.

To each vessel location (defined along an AIS track), one can associate a position uncertainty, depending on current condition, wind conditions, vessel manoeuvrability, and sounding position uncertainty. Based on the knowledge of:

- Vessel position
- Time
- Bathymetric information derived from vessel position,
- Tide information derived from vessel position and time thank to a given tide model,
- Vessel draft and possible heave

One can compute the worst case under keel clearance along the vessel trajectory. In a case grounding, the result will gives us:

- Grounding height $k$
- Ship speed ($S$)
- Sea floor type (rock, sand...)

Then, by computing UKC MinMax analysis over the entire existing marine traffic we can express:

- The grounding frequency over the study area:

$$F = \frac{\text{Number of Groundings}}{\text{Number of Vessels}}$$  \hspace{1cm} (27)

- The cost of these groundings:

$$C = \sum_{t=0}^{\infty} C_t(P_t, t_t)$$  \hspace{1cm} (29)
Where $n$ is the number of groundings, and $C_i(P_i, t_i)$ is the cost function associated to the grounding $i$.

Then the risk associated to the studying area will be:

$$ R = F \times C $$  \hspace{1cm} (31)$$

MinMax analysis does not provide a statistical model of the grounding risk as developed in [Calder, 2008]. It focuses on the “worst possible case”. It gives us a frequency and a cost of groundings. This analysis can easily be applied over a study area where AIS data traffic and complete hydrographic information are available. In addition, the risk definition in this analysis is semantically similar to the one from a risk matrix: a frequency computing with a consequence. We will develop this analysis in the risk matrix weights estimation process.

4.2.1.5. UKC model for risk matrix weights estimation

In this section, we present the different steps of the proposed weights estimation methodology using a UKC model. The main objective is to compute UKC model on different test area in order to compare its results with risk matrix ones. From this, we can objectively define the risk matrix coefficients by optimizing the risk matrix “expected results” versus the UKC MinMax result.

As we seen in the first section of this report, risk matrix dimension change from an area to another. Here we will focus on those which are invariant from a study to another:

- Marine traffic
- Sea-floor characteristics
- Hydrographic and cartographic state

**Marine traffic**

Marine traffic analysis is usually done by a studying AIS data. Lots of information can be compute from these data: ship type, position, speed, destination, time of departure and arrival. It had, for example, been used to study ship behaviours [Calder, 2009] according to type of navigation and ports characteristics.

In our study, a marine traffic simulation would need to match ship behaviours considering risk matrix situations: lack of bathymetric information, meteorological conditions, tide issues, navigation type, and port importance. A simulation on this scale will not be done here. We will compute UKC model on the existing marine traffic, by using AIS data traffic.

**Sea-floor characteristics**

Depth, shoal, sea-floor complexity influence risk result and ship behaviour. In addition to the difficulty of simulate a new bathymetric model, we would need to simulate a marine traffic either. As we seen in the previous section, modeling a marine traffic will not be done here. Another solution can be simulating shoals or rocks on existing bathymetric model with existing AIS dataset. By doing this, we involved subjectivity in our study: we choose the location of a shoal and so the numbers of groundings.

As the marine traffic issue, solution will be to use existing sea-floor which had been well surveyed.
Test zones
According to the previous sections, we need to find tests zones which will be representatives of:

- Marine traffic
- Type of navigation
- Sea-floor morphology

As an UKC model requires vessel trajectories and hydrographic information, we need to knows:

- AIS traffic data
- Full bathymetric coverage
- Tide model

![Figure 13: Example of possible test zones.](image)

This figure is an example of which zone will be interesting in our study. Zone 1 (at the left) represent a port approach situation, with an important marine traffic. Zone 2 (in the middle) is an open sea situation, with a deep sea floor and poor marine traffic density. Zone 3 (at the right) is representative of complex sea-floor and constraint navigation.

On each test zone, we can compute UKC model by varying the other criteria. We present in the next section how we can simulate these criteria and how to compute the UKC model.

Hydrographic survey simulation
The state of hydrographic data over an area is maybe one of the most important risk matrix dimensions. A lack of bathymetric information can generate poor visualization of navigational hazards, and poor sea bed morphology understanding. Ship routes and navigation margins may be biased by this lack of data. In this section we will try to express this risk by using UKC model on sparse bathymetric data cases.

The problem is how to design an estimator of the grounding frequency in simulating a lack of bathymetric data.
Let us consider a test zone, where a complete bathymetric model and so UKC model computation is possible. Here the risk matrix criterion of interest is the CATZOC attribute. This simulation can be divided in two parts:

- Data sampling
- Data interpolation

**Data sampling**

The sampling process can be done according to a hydrographic survey requirements associated to a specific CATZOC attribute. So, for each CATZOC we can divide it in two parts: survey lines design and instrument choice (or sounding accuracy). For the first two CATZOC classes (A1 and A2):

- CATZOC A1: Not simulated because our bathymetric model already has CATZOC A1 attributes: full coverage, multi-beam echo-sounder system.
- CATZOC A2: No needs of data sampling here, but we decrease sounding accuracy

For the other CATZOC classes we need to sample the existing information and so simulate hydrographic surveys. Instrument used will be a single beam echo-sounder (SBES) in order to create a partial coverage. The line design can follows two definitions: IHO standards for hydrographic surveys S-44 and a function of nautical scale. We will use these two definitions as hereafter:

- CATZOC B: Line spacing defined by the S-44 order 1b.
- CATZOC C: Line spacing as a function of nautical scale
- CATZOC D: Line spacing as a function of nautical scale, but we simulate a virtual increasing of the nautical scale (a Port approach scale became a coastal scale).
- CATZOC U: Same as CATZOC D, instrument choices: leadline.

![Figure 14: example of data sampling according to a hydrographic survey design](image-url)
Data interpolation

After the sampling step, we need to interpolate these new data in order to create a new bathymetric model. This interpolation needs two important characteristics:

1. Reflect the navigator visualization of the sea floor according to the sampling data.
2. Remain between the minimum and maximum of sampling soundings. It must not create any features (shoals for example) which do not exist.

Notice that the interpolate process will create additional modeling errors and thus, additional uncertainty.

Let us note:

- $\Omega_E$: set of entries zone soundings
- $\Omega_S$: set of sampling soundings
- $\Omega_I$: set of interpolate soundings

We have:

$$\Omega_S \subset \Omega_E$$

So

$$\text{Min}(\Omega_S) \geq \text{Min}(\Omega_E)$$

And, the second interpolation characteristic gives us:

$$\text{Min}(\Omega_I) \geq \text{Min}(\Omega_S)$$

By sampling and interpolate the existing bathymetric data, the minimum interpolated sounding will be deeper or equal than the actual one. The next figure expresses the difference.

![Sampling and interpolation error along a ship trajectory](image)

Figure 15: Sampling and interpolation error along a ship trajectory

By induction, the minimum of the under keel clearance along the ship trajectory will be higher with interpolate data, and so the risk lower. Let us note $\text{Min}(UKC_{\text{True}})$ the under keel clearance
minimum using the actual bathymetric data and \( \text{Min(UKC}_{\text{thought}}) \) the one using the interpolate data.

![Graph](image1.png)

**Figure 16:** Under keel clearance along a ship trajectory.

We consider that the minimum under keel clearance result \( \text{Min(UKC}_{\text{True}}) \) is the navigational clearance margin chooses by the navigator. We consider that the navigator will took the same margin in a case of a lack of data. Therefore, we will subtract the two minimums difference from the UKC model (it can represent a higher draft or a difference in the tide) in order to get the same navigation margin on UKC model computation. Grounding event will be true if the difference is higher than the actual under-keel clearance along the vessel track.

\[
\text{Grounding event is true if: } \text{UKC} - \left( \text{Min(UKC}_{\text{thought}}) - \text{Min(UKC}_{\text{True}}) \right) < 0
\]

![Graph](image2.png)

**Figure 17:** Lack of data risk. A grounding event will be true if the difference between the true and thought under-keel clearance is higher than the actual under-keel clearance minimum.
Sampling bathymetric data will not always imply a grounding event. We need to follow a Monte Carlo method: simulate a lot of hydrographic survey. It will allow us to get the impact of hydrographic data state on grounding frequency.

In addition to this interpolation error, we need to take care of possible cartographic error. In particular horizontal chart shifts. Some charts present coastlines which are compute from old datum and technology. Then these charts are rasterising and use in ENC with GNSS positioning system. This error can easily be expressed and simulated in our model by a horizontal offset and position uncertainty.

**Currents and meteorological conditions**

Currents and winds influence ship handling and trajectory. It could be a cause of grounding, for example drift vessel to a shoal, or out of entrance channel.

In our simulation these meteorological conditions will be expressed by an increase of the position uncertainty ellipse size. It can be defined by using guidelines for marine corridors design [Canadian Coast Guard, 2001]. Indeed, marine corridors sizes are designed to permit ships to be able to prevent any risk in different cases. According to the Canadian Coast Guard report, we will define the uncertainty ellipse length by:

<table>
<thead>
<tr>
<th>Ship Type</th>
<th>Handling</th>
<th>Handling Coefficient</th>
<th>Room for manoeuvre length</th>
</tr>
</thead>
<tbody>
<tr>
<td>War vessel, Victory cargos</td>
<td>Excellent</td>
<td>1.6</td>
<td>1.6*Ship Width</td>
</tr>
<tr>
<td>Tankers, New ore carriers, Liberty cargo</td>
<td>Good</td>
<td>1.8</td>
<td>1.8*Ship Width</td>
</tr>
<tr>
<td>Old ore carriers, damage ships</td>
<td>Poor</td>
<td>2.0</td>
<td>2.0*Ship Width</td>
</tr>
</tbody>
</table>

**Table 6: Room for manoeuvre length according to ship type**

Then, we add the wind and current contribution to the position uncertainty:

<table>
<thead>
<tr>
<th>Wind intensity</th>
<th>Excellent</th>
<th>Good</th>
<th>Poor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.0*Ship Width</td>
<td>0.0*Ship Width</td>
<td>0.0*Ship Width</td>
</tr>
<tr>
<td>Moderate</td>
<td>0.3*Ship Width</td>
<td>0.4*Ship Width</td>
<td>0.5*Ship Width</td>
</tr>
<tr>
<td>Strong</td>
<td>0.6*Ship Width</td>
<td>0.8*Ship Width</td>
<td>1.0*Ship Width</td>
</tr>
</tbody>
</table>

**Table 7: Wind contribution in position uncertainty according to wind strength**

And the current contribution:

<table>
<thead>
<tr>
<th>Current intensity</th>
<th>Excellent</th>
<th>Good</th>
<th>Poor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not significant (&lt;0.2 knts)</td>
<td>0.0*Ship Width</td>
<td>0.0*Ship Width</td>
<td>0.0*Ship Width</td>
</tr>
<tr>
<td>Light (0.2-0.5 knts)</td>
<td>0.1*Ship Width</td>
<td>0.2*Ship Width</td>
<td>0.3*Ship Width</td>
</tr>
<tr>
<td>Moderate (0.5-1.5 knts)</td>
<td>0.5*Ship Width</td>
<td>0.7*Ship Width</td>
<td>1.0*Ship Width</td>
</tr>
<tr>
<td>Strong (&gt;1.5 knts)</td>
<td>0.7*Ship Width</td>
<td>1.0*Ship Width</td>
<td>1.3*Ship Width</td>
</tr>
</tbody>
</table>

**Table 8: Current contribution on ship position uncertainty according to current strength**
Then, by computing these three tables, we will be able to define the position uncertainty due to wind and currents conditions.

### 4.2.2. Conclusion on weights estimation

By computing the UKC model on different situations, we would be able to express the influence of the risk matrix likelihood components on grounding frequency. These components can be divided into two groups: the ones that can't be simulated and the ones that can. For the first group, test zones need to be representative of all possible scenarios to estimate the most general weights. Notice that if a scenario is not present in the entire studying area, the risk matrix weights will be more accurate if the weight estimation process does not take care of the missing scenarios.

The second group’s criteria will be simulated. These simulations must match up the reality. The more criteria we would be able to simulate, the less actual data we will need and therefore the more scenarios we will be able to study.

Let us resume the different steps of the presented weights estimation:

![Figure 18: Weights Estimation Process](image)

- **Test Zones Identification:**
  - Presence of:
    - Vessel tracks
    - Full Bathymetric coverage
    - Tidal Model
  - Representatives of:
    - Type of Navigation
    - Sea-Floor Morphology

- **Objective Risk Model Computation:**
  - According to:
    - Vessel tracks
    - Bathymetric model
    - Tidal Model

- **Test Zone Characteristics Simulation:**
  - Hydrographic surveys
  - Meteorological Conditions
  - Bottom Type

- **Weights Estimation:**
  - UKC Model Results Comparison

The weights estimation is an iterative process and can be adapted to the type of risk matrix or to the available test data sets. It also can be used in order to studying the impact of a hydrographic survey on grounding frequency. In the same way, we would be able to study the impact of a...
horizontal chart datum or tidal model improvement on the grounding frequency. This kind of information would be helpful in the prioritization process. It can improve the Level of Effort computation.

5. **Risk Assessment method recommendations**

This report regroups a lot of different studies on risk assessment methods for hydrographic survey prioritization purposes. We will here proposed a possible process, based on FSA steps, to prioritize survey areas.

5.1. **Hazard identification.**

This step defines the different hazards and goals of our study. All stakeholders needs to be involves in the hazard identification. Here is a proposed list of these stakeholders:

- CHS experts
- Economists experts
- Environmental experts
- Arctic navigators
- Communities representatives
- Ports authorities
- Mining and oil and gas terminal projects managers

This working group will define:

- Arctic navigational issues and hazards.
- Actual and predicted economic stage in the Arctic.
- Politics issues.
- Environmental issues.

5.2. **Risk analysis.**

We had present different risk analysis method for hydrographic surveys prioritization purposes. We will notice the most two important concepts:

- Weighted overlay analysis
- Risk Matrix concept

The different step to follow will be:

- Define a risk definition.
- Risk matrix criteria definition.
- Risk Matrix Weights estimation.
- Risk matrix computation.
5.3. **Cost/Benefit assessment of these risk control options.**

This step need to get a good overview of the different actual or in project economic sites of importance. In addition, economic expert needs to assess the impact of grounding on the local economy and compare it to the cost of an hydrographic survey. A predictive study also can assess the cost of an un-surveyed zone, considering the grounding probability, and compare it to the cost a hydrographic survey on this same zone.

5.4. **Recommendation for decision making process.**

The final step will gather all available information in order to prioritize the different survey areas. The two most important data being the risk map from the risk matrix, and the cost/benefit study results.

Prioritization process can rely on different issues: economic, politic, environmental, financial. In the Arctic case, it would be interesting to assess the different period where an hydrographic survey is possible. Indeed, due to weather and Ice issue, some areas are close to surveying.
6. Conclusion

Risk assessment for hydrographic surveys prioritization is a more and more studied topic. Lots of hydrographic services stared research on it. It also regroup a lots of different issues:

- Risk assessment methods
- The notion of Hydrographic risk
- Traffic analysis
- Data compilation
- Economic studies

In this report we had explain three approaches that have been developed on this subject. They all used GIS tools in order to compute different kind of risk data. But their risk definition and prioritization process diverged.

We had shown that the ARHC and CHS risk notion is more oriented towards chart adequacy to a given maritime traffic. By focusing their study on bathymetric issues they get a quasi objective model, but they miss lots of grounding causes.

The LINZ study follows an existing marine risk assessment process (FSA), and clearly defines risk as the computation of a frequency and a consequence. Their risk matrix is defined in order to match this definition. All criteria are defined, documented, marked, and weighted. The computation of this matrix is done in the same way as the ARHC and CHS GIS tools. But, due to the large number of criteria, they need to quantify the importance of each criterion in the model. By doing so, they involve subjectivity in their process. It raised the issue of weights estimation.

Weight estimation need to represent the criteria importance in the risk matrix. It can be done by using historical grounding data and study the type of traffic involved, which were the causes and consequences. But by doing so, we do not have any a predictive data. In addition, we had shown that we need all possible risk matrix scenarii to get the most representative weights. The solution presented is to use a mathematical an objective grounding model and apply it on different scenarii.

The risk model presented in this report can be used only on area where there is a complete knowledge of the sea-bed. Then some criteria can be simulated, when some other can not. This model is not meant to replace the risk matrix. It is not general enough and it does not compute consequence criteria weights. But it can be improved. One problem of this model is the marine traffic. We are using AIS data traffic for now, and we study the impact of the CATZOC attribute by assuming that the navigator will take the same navigation margin. We could imagine a model which will change ships trajectories when we change the CATZOC attribute based on the new bathymetric model and the risk taken by the navigator.
Bibliography


Appendix A: Under-Keel clearance uncertainty model

Sounding error

In this section we will define the sounding term and its error components. Let us note \( P_0 = (E_0, N_0) \) the position, \( Z(P_0) \) the assumed sounding at the position \( P_0 \), \( Z_v(P_0) \) the true sounding at the position \( P_0 \) and \( \delta Z_v(P_0) \) the sounding uncertainty. We have:

\[
Z(P_0) = Z_v(P_0) + \delta Z_v(P_0) \tag{A.1}
\]

At a position \( P_0 + \delta P \) and at order one in \( \delta P \), we have:

\[
Z(P_0 + \delta P) = Z_v(P_0) + \delta Z_v(P_0) + \frac{dZ}{dP}(P_0) \delta P
\]

\[
= Z_v(P_0) + \delta Z_v(P_0) + \frac{dZ_v}{dP}(P_0) \delta P + \frac{d\delta Z_v}{dP}(P_0) \delta P \tag{A.2}
\]

Where

- \( \frac{dZ_v}{dP}(P_0) \) is the local slope of the sea-floor
- \( \frac{dZ_v}{dP}(P_0) \delta P \) define the error due to the position uncertainty
- \( \frac{d\delta Z_v}{dP}(P_0) \) is the local variation of the sounding uncertainty. Assuming that the sounding error is be the same over a small area, this component is equal to zero.

Then

\[
Z(P_0 + \delta P) = Z_v(P_0) + \delta Z_v(P_0) + \frac{dZ_v}{dP}(P_0) \delta P \tag{A.3}
\]

Where \( \frac{dZ_v}{dP}(P_0) \) is the gradient of the sea-floor at the position \( P_0 \) defined by:

\[
\frac{dZ_v}{dP}(P_0) = \begin{bmatrix} \frac{\partial Z_v}{\partial E}(E_0) & \frac{\partial Z_v}{\partial N}(N_0) \end{bmatrix} \tag{A.4}
\]

And

\[
\frac{dZ_v}{dP}(P_0) \delta P = \frac{\partial Z_v}{\partial E}(E_0) \delta E + \frac{\partial Z_v}{\partial N}(N_0) \delta E \tag{A.5}
\]


**Tide error**

Following the same lines, we will define the tide error. Let us write \( P_0 = (E_0, N_0) \) the position, \( t_0 \) the time, \( T(P_0, t_0) \) the assumed model tide at position \( P_0 \) and time \( t_0 \), \( T_v(P_0, t_0) \) the true tide at position \( P_0 \) and time \( t_0 \) and \( \delta T(P_0, t_0) \) the tide uncertainty. We have:

\[
T(P_0, t_0) = T_v(P_0, t_0) + \delta T(P_0, t_0)
\]  

(A.6)

At a position \( P_0 + \delta P \) and at a time \( t_0 + \delta t \) we have:

\[
T(P_0 + \delta P, t_0 + \delta t) = T_v(P_0, t_0) + \delta T(P_0, t_0) + \frac{dT}{dP}(P_0)\delta P + \frac{dT}{dt}(t_0)\delta t
\]

\[
T(P_0 + \delta P, t_0 + \delta t) = T_v(P_0, t_0) + \delta T(P_0, t_0) + \frac{dT_v}{dP}(P_0)\delta P + \frac{d\delta T}{dP}(P_0)\delta P + \frac{dT_v}{dt}(t_0)\delta t + \frac{d\delta T}{dt}(t_0)\delta P
\]  

(A.7)

Where
- \( \frac{dT_v}{dP}(P_0) \) is the tide gradient with respect to the position. We will consider that the tide variations are insignificant for a small position variation \( \delta P \).
- \( \frac{d\delta T}{dP}(P_0) \) is the local variation of the tide uncertainty versus position. We will assume that tide’s error will be the same in a given small area, so this component is supposed to be zero.
- \( \frac{dT_v}{dt}(t_0) \) is the tide variation versus time. This derivative will not be significant.

Following these assumptions and simplifications, the water level variation can be written by:

\[
T(P_0 + \delta P, t_0 + \delta t) = T_v(P_0, t_0) + \delta T(P_0, t_0)
\]  

(A.8)

The term \( \delta T(P_0, t_0) \) may include two types of errors:
- The first one is may be a tidal measurement error, due to instrument and Chart Datum (CD) uncertainties. CD uncertainty may be due to a too short time series used for its estimation.
- The second one may be due to the fact that the tide model used at vessel’s position \( P_0 \) may be defined for another position \( P_1 \). Consequently, the term \( \delta T(P_0, t_0) \) may include the spatial variation of tide due to the use of a model (or tide gauge) at a remote location.
Other components

Other component of the UKC uncertainty model do not depend of time (we may consider that heave is a centered random variable) and vessel’s position. But they may be submitted to measurement errors or uncertainties. We will represent these errors by:

\[ d = d_v + \delta d \]
\[ h = h_v + \delta h \]

All these components can be seen as centered variables and their uncertainties as constant offsets.

We can now propagate the uncertainty due to all component of the UKC equation in a global linearized model:

\[ u(P, t) = u(P_0 + \delta P, t_0 + \delta t) = Z_v(P_0) + T_v(P_0, t_0) + (d_v + h_v) + \delta u \] (A.9)

Where

- \[ \delta u = \delta Z(P_0 + \delta P) + \delta T(P_0 + \delta P, t_0 + \delta t) - (\delta d + \delta h) \]
- \[ \delta Z(P) = \delta Z_v(P_0) + \frac{\partial Z_v}{\partial P}(P_0) \delta P \]
- \[ \delta T(P, t) = \delta T(P_0, t_0) \]

Under-keel clearance uncertainty model

We can now propagate uncertainties through equation (A.9).

\[ \sigma_{ukc}^2 = \frac{\partial Z_v}{\partial E}(E_0)^2 \sigma_{\delta E}^2 + \frac{\partial Z_v}{\partial N}(N_0)^2 \sigma_{\delta N}^2 + \sigma_{Z(P)}^2 + \sigma_{\delta T(P)}^2 + \sigma_{h}^2 \] (A.10)

Where: \[ \sigma_{\delta T(P,t)}^2 = \sigma_{T(P_0,t_0)}^2 \]

We shall denote by \( \sigma_x^2 \) the variance of a random variable \( x \).

From equation (A.10), we see that whenever all source of error follows a normal law, the UKC will also follow a normal law which variance depends on:

- \( \left( \frac{\partial Z_v}{\partial E} \right)^2 \sigma_{\delta E}^2 + \left( \frac{\partial Z_v}{\partial N} \right)^2 \sigma_{\delta N}^2 \) : a term which depends on the expected or estimated terrain morphology. It should be noticed that this term can only be defined locally, in other words, terrain morphology can’t be extrapolated for significant positional errors.
- \( \sigma_{Z(P)}^2 \) : sounding uncertainty;
- \( \sigma_{T(P_0,t_0)}^2 \) : tide uncertainty;
- \( \sigma_d^2 \) : draft uncertainty;
- \( \sigma_h^2 \) : heave uncertainty.

In summary, if the sounding, tide, draft, and heave uncertainty follows a normal law, then the UKC also follows a normal law, which is described by its most probable estimate at time \( t \)

\[ u(P, t) = Z_{\text{obs}}(P) + T_{\text{obs}}(P, t) - (d_{\text{obs}} + h_{\text{obs}}(t)) \] (A.11)
Appendix B: Weights Estimation

We will here describe the weights estimation equation.

Let us remind the risk matrix computation:

\[ r = (X_T \cdot W_T) \cdot (X_L \cdot W_L) \cdot (X_C \cdot W_C) \]  \hspace{1cm} (A.1)

Where:

- \( X_T, X_L, X_C \) are the matrix marks for Traffic, Likelihood and Consequence sections.
- \( W_T, W_L, W_C \) are the matrix weights for Traffic, Likelihood and Consequence sections.
- \( r \): Risk matrix result

The risk result is a combination of marks and weights. Notice that the equation is not linear if we tried to estimate all weights. In our case, we will only estimate the likelihood weights, and so the equation (B.1) became linear.

Considering one risk matrix scenario. To estimate likelihood weights, we need to minimize the following difference:

\[ \text{Min} \left( \frac{r}{(X_T \cdot W_T) \cdot (X_C \cdot W_C)} - (X_L \cdot W_L) \right)^2 \]  \hspace{1cm} (A.2)

Then, for a number \( j \) of different scenarios:

\[ \text{Min} \left( \sum_{j=0}^{m} \left( R_j - \sum_{i=0}^{n} W_i \cdot X_{ij} \right)^2 \right) \]  \hspace{1cm} (A.3)

Where:

- \( m \): Number of scenarios
- \( n \): Number of likelihood criteria

\[ R = \frac{r}{(X_T \cdot W_T) \cdot (X_C \cdot W_C)} \]  \hspace{1cm} (A.4)

Considering that the sum of all weights need to be equal at one, our estimation will need to answer these two conditions:
We are here in a constraint optimization situation. We will use the Lagrangian process. Let us define:

$$\mathcal{L}(W, \lambda) = \sum_{j=0}^{m} \left( r_j - \sum_{i=0}^{n} W_i * X_{ij} \right)^2 - \lambda \left( \sum_{i=0}^{n} W_i - 1 \right)$$  \hspace{1cm} (A.6)

Then, by deriving the equation (B.6) per the weight $W_p$, we obtain:

$$\frac{\partial \mathcal{L}(W, \lambda)}{\partial W_p} = -2 \sum_{j=0}^{m} \left( r_j - \sum_{i=0}^{n} W_i * X_{ij} \right) - \lambda$$  \hspace{1cm} (A.7)

In order to optimize weights, we need for each $W_i$ find:

$$\frac{\partial \mathcal{L}(W, \lambda)}{\partial W_i} = 0$$  \hspace{1cm} (A.8)

So we obtain the two equations:

$$\begin{cases} 
\sum_{j=0}^{m} \left( X_{pj} * \sum_{i=0}^{n} W_i * X_{ij} \right) - \frac{\lambda}{2} = \sum_{i=0}^{n} \left( W_i * \sum_{j=0}^{m} X_{pj} * X_{ij} \right) - \frac{\lambda}{2} = 0 \\
\sum_{i=0}^{n} W_i = 1 
\end{cases}$$  \hspace{1cm} (A.9)

By deduction, we can obtain the matrix form of (B.9):

$$\begin{pmatrix} 
\sum_{j} X_{1j}^2 & \ldots & \sum_{j} X_{1j} * X_{nj} & -\frac{1}{2} \\
\vdots & \ddots & \vdots & \vdots \\
\sum_{j} X_{nj} * X_{1j} & \ldots & \sum_{j} X_{nj}^2 & -\frac{1}{2} \\
\frac{1}{2} & \ldots & \frac{1}{2} & 0 
\end{pmatrix} \begin{pmatrix} 
\sum_{j} X_{1j} * r_j \\
W_i \\
\sum_{j} X_{nj} * r_j \\
\frac{1}{\lambda}
\end{pmatrix} = \begin{pmatrix} 
\sum_{j} X_{1j} * r_j \\
\sum_{i} W_i \\
\sum_{j} X_{nj} * r_j \\
\frac{1}{\lambda}
\end{pmatrix}$$  \hspace{1cm} (A.10)
With:

\( \tilde{W}_i \): The estimate weight \( W_i \)
\( \tilde{\lambda} \): The estimated Lagrangian factor.

The equation (B.10) can be seen as:

\[
A \ast X = B
\]  \hspace{1cm} (A.11)

Then the \( X \) vector estimation can be done with a simple Least Square Estimation.